Polynomials – Problems

1. Simplify the following:

(a) $ab^2 + a^2b - a^2b^2 + 2ab^2 - 2a^2b + a^2b^2$

(b) $[2x(x-5) + 3x^2(5-x)] \div [x^2 - 5x + 6]$

(c) $\dfrac{x^3 - 3x^2 - 4x + 12}{x - 3}$

2. Factor and find the zeros, if possible:

(a) $5x - 15$

(b) $2x^3y - 6x^2y^5$

(c) $x^2 - 8x$

(d) $x^2 + 7x + 12$

(e) $x^2 + 5x - 14$

(f) $x^2 - 13x + 40$

(g) $x^2 - 25$

(h) $4x^2 - 9y^2$

(i) $2x^2 - 9x - 5$

(j) $4x^2 + x - 5$

(k) $9a^2 - 1$

(l) $xy - 3x + 2y - 6$

(m) $x^2 + 3xy + 2y^2$

(n) $3ab^4 - 12a^2b^3c + 36b^5c^4$

(o) $100a^4b^2 - 36c^{10}$

3. How many distinct roots does each of the quadratic polynomials have?

(a) $y = x^2 - 3x + 5$

(b) $y = x^2 + 3x - 5$

(c) $y = x^2 + 2\sqrt{2}x + 2$

(d) $y = x^2 - 9x - 10$

4. Give an example of each of the following polynomials, if such a polynomial exists. If not, state why not.

(a) A quadratic with roots $x = 1, -3$.

(b) A degree 3 polynomial with roots $x = 1, -3$.

(c) A factor of $x^4 - 61x^2 + 900$.

(d) Three different quadratic polynomials with roots $x = 1, 2$.

5. Perform the following polynomial division:

(a) $\dfrac{x^3 + 2x^2 - x - 2}{x + 2}$

(b) $(2x^3 + 3x^2 - 8x - 12) \div (x^2 - 4)$

(c) $\dfrac{3x^2 + 2x - 1}{5x + 1}$

(d) $\dfrac{x^9 - 1}{x - 1}$

(e) $\dfrac{x^n - 1}{x - 1}$ for any positive integer $n \geq 2$