

## Functions and Inverses – Problems

- If  $f(x)$  is an invertible function and  $f(2) = -5$ , what is  $f^{-1}(-5)$ ?
  - If  $f(x)$  is an invertible function and  $f(0) = 2$ , what is  $f\left(f^{-1}(0)\right)$ ?
  - Let  $f(x) = x^3$ . At how many points do the graphs of  $y = f(x)$  and  $y = f^{-1}(x)$  intersect?
- Find the domain of the following functions:
  - $f(x) = \frac{\sqrt{2x+7}}{x-|x|}$
  - $f(x) = \frac{3}{3x-x^2} + \frac{\sqrt{9-x^2}}{3}$
- Let  $f(x) = \frac{7x+1}{9-2x}$ . Find  $f^{-1}(x)$ .
  - Does the relation described by  $xy = 7y+8$  define a function in the variable  $y$ ? What about a function in the variable  $x$ ?
  - Under which circumstances does the function  $f(n) = n!$  ( $n$  factorial) have an inverse function? Where doesn't the inverse exist? Explain.
  - Let  $h(x) = x^2 - 2x + 8$  and  $g(x) = \sqrt{x}$ . Write an expression for  $(g \circ h \circ g)(x)$  in terms of  $x$ .
  - Let  $f(x) = x + 1$  and  $g(x) = \frac{1}{x}$ . Compute  $(f \circ g)^{-1}(2)$ .
- Which of the following relations are functions of  $q$ :

$$w = q + 1 \quad , \quad q = \frac{2w01}{w} \quad , \quad wq = -27.$$

- Find the inverse function  $f^{-1}$  for each of the following.

$$f(x) = 3x + 2 \quad , \quad x^2 + 6x + 3 \text{ for } x \leq -3 \quad , \quad f(x) = \frac{x+3}{5x-1}.$$

- Let  $f(x) = \frac{2x}{1-x}$ . Find all real numbers  $x$ , if any, for which  $f(-x) = 2f(x)$ .
- Let  $f(x) = \sqrt{x} + 1$ ,  $g(x) = x^2 - x$ , and  $h(x) = \frac{1}{x-2}$ . Evaluate and simplify the following

$$f\left(g(x)\right) \quad , \quad (h \circ g)(x) \quad , \quad f\left(g(h(x))\right) \quad , \quad (g \circ h \circ g)(x).$$

5. Give a different function for each of the following questions so that the function has exactly the given domain and range.
- (a) Domain =  $\mathbb{R}$ , Range =  $\mathbb{R}$ .
- (b) Domain =  $(-\infty, 0) \cup (0, \infty)$ , Range =  $(-\infty, 0) \cup (0, \infty)$ .
- (c) Domain =  $\mathbb{R}$ , Range =  $\{4\}$ .
- (d) Domain =  $(0, 1) \cup (1, 2) \cup (2, \infty)$ , Range =  $(3, \infty)$ .

## Exponentials and Logarithms

1. (a) If  $f(x) = 2^x$ , then what is  $f^{-1}(1024)$ ?  
 (b) Find the inverse function  $f^{-1}(x)$  of  $f(x) = e^{4x-2}$ .
2.  $\frac{e^{7x-1}}{e^{x-1}} = (e^6)^7$ . Find  $x$ .
3. For how many values does  $e^x = 0$ ? What does that tell us about the value of  $\ln(0)$ ?
4. Evaluate the following

$$\log_3 \left( \frac{1}{27} \right) \quad , \quad \log_{\frac{1}{2}} \left( \frac{1}{4} \right) \quad , \quad \log_{25} (\sqrt{5}) \quad , \quad \ln(1).$$

5. Are there any solutions to the equation

$$\ln(x^3 - 2x^2 - x + 2) - \ln(x + 1) - \ln(x - 2) = -\ln(2)?$$

Why or why not?

6. Evaluate the following.

(a)  $\frac{e^{1+2 \ln 7}^2}{(7^{1+\ln \sqrt{7}})^2 (7^3)^{1+\ln 7}}$

(b)  $\frac{\log_5 25 - \log \frac{1}{10}}{3^{\log_3 2} - e^{\ln 8}}$

(c)  $\frac{\log_6 4 - 2 \log 25(5) + \log_6 9}{\log_5 (3^{-1} - 3^{-2} - 27^{-1})}$

7. Solve the following equation:

$$e^{\ln x + \ln(x+4)} = 5.$$

8. Do the graphs of  $y = e^x$  and  $y = \ln x$  intersect? If so, where? If not, how do you know?

9. Which of the following are equal to  $\frac{1}{2}$ :

$$e^{\ln(0.5)} \quad , \quad e^{-\ln(2)} \quad , \quad \ln(1) - \ln(2) \quad , \quad \frac{3e^0}{6} \quad ?$$

Write the equation of the horizontal asymptote to the graph  $y = e^x$  and the equation of the vertical asymptote to the graph of  $y = \ln x$ . Use the definition of “inverse functions” to explain how the equations of the two asymptotes are related.

10. Solve the following equation:

$$2^x = 5^{9x-2}.$$

11. If the population of rabbits on a particular island is given by the equation  $P = 10 \cdot 2^t$ , where  $t$  is the time (in year), find the initial population of rabbits on the island.

Then find how many years it will take for the population to reach 1000.