Logic – Problems

1. For each of the following statements, determine if they are true or false.

(a) If x is a positive integer such that $x - 1 < 3$, then $x \leq 4$.

(b) For every real number x there exists a real number y such that $x + y = 0$.

(c) If x is a non-zero rational number, then $\frac{1}{x} < x$.

(d) If x is a real number which satisfies $x^2 + 1 = 0$, then every real number is rational.

2. For each of the following statements, identify the hypothesis and the conclusion.

(a) If x is a positive real number that satisfies $x^2 - 2x + 1 < 0$, then $x < 1$.

(b) If x is a real number, then either $x \leq 0$, $x \geq 0$.

(c) If there exists a real number x such that $x^4 + 1 = 0$, then any real number is either an integer or a multiple of π.

(d) The inverse of a irrational number is also irrational. (Hint: Start by rewriting this as an "If..., then..." statement.)

3. For each of the following statements, find the opposite (or negation).

(a) If x is not rational, then for every integer n, $x + n$ is not rational.

(b) If x is positive and rational, then $\frac{1}{x}$ is positive and rational.

(c) The inverse of a irrational number is also irrational. (Hint: Start by rewriting this as an "If..., then..." statement.)

(d) For any real number x, either $x \leq 0$, $x \geq 0$.

4. For each of the following "if and only if" statements rewrite it as a pair of "if..., then..." statements.

(a) x is rational if and only if $x + 5$ is rational

(b) A positive integer x is even if and only if $x + 1$ is odd.

(c) A rational number x is positive if and only if $-x$ is negative.

(d) A real number x is not rational if and only if for every integer n, $x + n$ is not rational.

5. Let $\{a_n\}$ be a sequence of real numbers: $a_1, a_2, a_3, \ldots$

$\{a_n\}$ is said to be **alternating** if for any $n \in \mathbb{N}$, $a_n \cdot a_{n+1} < 0$.

Using **words**, write the statement "$\{a_n\}$ is not alternating.".

6. Let $\{a_n\}$ be a sequence of real numbers: $a_1, a_2, a_3, \ldots$

$\{a_n\}$ is said to be **decreasing** if for any $n \in \mathbb{N}$, $a_{n+1} < a_n$.

Using **words**, write the statement "$\{a_n\}$ is not decreasing.".

7. Consider the statement: $R =$ " For all natural numbers n, n is even or $n^2 - 1$ is divisible by 4 ".

(a) Write the statement R using the **logical symbols.**

(b) Is the statement R **TRUE** or **FALSE** ?

8. Consider the statement: $R =$ "For all real numbers x , $(x-6)^2 = 4$ implies $x = 8$" .

(a) Write the statement R using the **logical symbols.**

(b) Is the statement R **TRUE** or **FALSE** ?

9. For each statement, decide whether they are TRUE or FALSE. Explain your decision briefly.

(a) "For all x, there exists y such that $x < y$.

(b) "There exists x such that for all y, $x < y$.

(c) "For all x and for all y, $x < y$.

(d) "There exists x and there exists y such that $x < y$.

(e) "There is a smallest positive number".

(f) "Every rational is a product of two rationals".

(g) "The equation $x^2 + y^2 = 4$ has a solution (x, y) in which both x and y are natural numbers".

(h) "The equation $x^2 + y^2 = 25$ has a solution (x, y) in which both x and y are natural numbers".

(i) "Every real number can be written as a difference of two positive real numbers".